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SEVERAL WORKS ON DISTRIBUTION OF DEPARTURES

Abstract: The curves of density function inspire to take many extensions of it. This paper is the extension work of the empirical truncated probability distribution on departures. From the attracting characteristics of the distribution we made some theoretical properties of the distribution by using graph of the density function.

Key Words: Random variable, departure rate, probability density function, probability distribution, empirical probability distribution, queuing system.

Introduction This paper is continuous work to the previous ones [2], which briefed here. The Random variable of interest is to "departing only (N-n) persons from the system". Instead of asking "In fixed time how many departures will take place?", We ask how likely the interval to have 'N-n' fixed no. of departures out of N arrivals. By taking inconsiderable subtle transformations on the existing distributions several new distributions have been developed. Since X is continuous, the PDF should be a function. We had made some inferences about this unknown function. This means the probability distribution that takes into account of measurements those we have surveyed for a considerable period of time. So the output of the inference problem is some distribution of X. We charted the histograms for different number of departures from the system from which we found the density curves.

In which case its probability density function is given by

$$f(x) = \begin{cases} \dfrac{e^{-\mu x}(\mu x)^{N-n}\mu}{(N-n)!} & for\ n \geq 0 \\ 0 & otherwise \end{cases}$$

where N is the restricted number of arrivals to the system.

μ Is the departure rate.

n is no. of persons remained in the system after taking (N-n) persons their service.

Graph of density function:

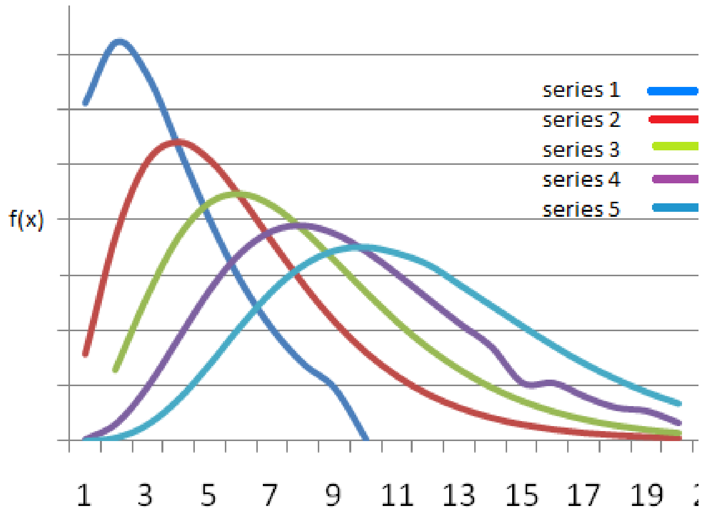

Figure - 1

Series 1 represents the probability density function graph when n = 0.

Series 2 represents the probability density function graph when n = 1.

Series 3 represents the probability density function graph of when n = 2.

Series 4 represents the probability density function graph when n = 3.

Series 5 represents the probability density function graph when n = 4.

X- Axis represents time; Y- represents f(x).

By observing the density function graphs, we would note that each series peaks at a particular point and then stars to descend. That is each series had peak point in particular time interval and then starts descending then running parallel to x-axis.

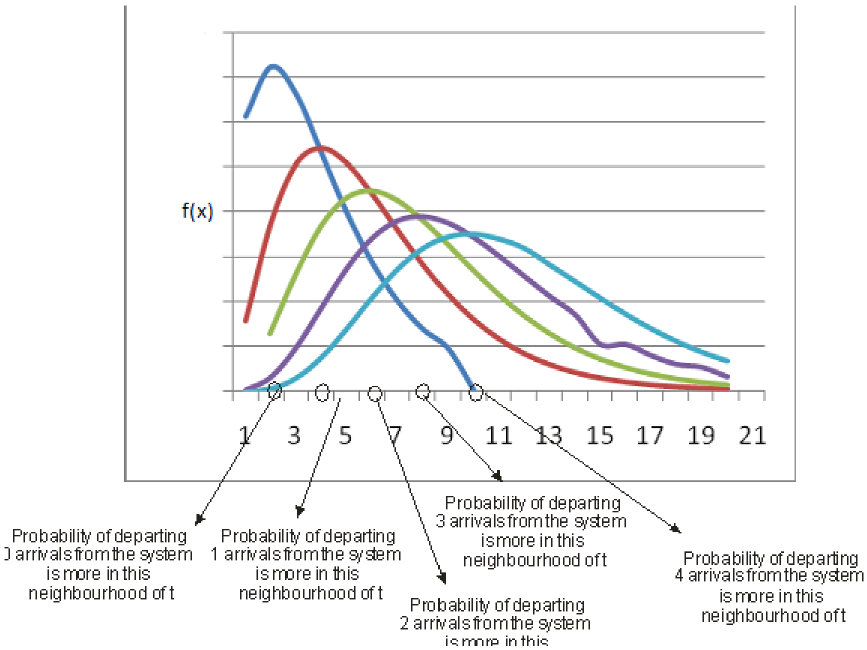

Figure 2

Graph of density function

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X- Axis represents time; Y- represents f(x).

A short survey was conducted in a bank queuing system to get the service rate. It is observed that an average of 0.51 customers depart per second. The system starts with N customers at time ‘0’ and no customers are allowed in to the queuing system. Let us take N=6 customers. If 1 customer had taken the service, then the no. of people remained in the system is n=5.

We have taken graph for (N-1), (N-2), (N-3), (N-4), (N-5) customers remained in the queuing system.

Here for N = 6,

N = (6-1), (6-2), (6-3), (6-4), (6-5)

i.e n = 5, 4, 3, 2, 1.

For n = (N-1) $\Rightarrow$ i.e. n = 6-1 = 5

For n = 5 we have crest point at around t = 2 Seconds.

t = 2 seconds. This can be written as

t = 2(1) seconds

t = 2 (the no. of customers who received the service) seconds.

t = 2(a) seconds.

Where a is the number of customers who received the service.

For n = N-2 $\Rightarrow$ n = 6-2 = 4

For n = 4 we have crest point around t = 4 seconds

t = 4 seconds. This can be written as

t = 2 (2) seconds

t = 2 (The number of customers who received service) seconds.

t = 2(a)

Where a is the number of customers who received the service.

For n = N-3 $\Rightarrow$ n = 6-3 = 3

For n=3 we have crest point at t = 6 seconds

That can be written as

t = 6 seconds

t = 2(3) seconds

t = 2 (The no. of customers who departed from the system) second

t = 2(a) seconds

Where a is the number of customers who received the service.

For n = N – 4 $\Rightarrow$ 6 – 4 = 2

For n = 2 we have crest point at t = 8 seconds that 8 seconds can be split as

t = 8 seconds

t = 2(4) seconds

t = 2 (No. of customers who departed from the system seconds)

t = 2 (a) seconds

Where a is the number of customers who received the service.

For n = N – 5 $\Rightarrow$ 6 – 5 = 1

For n = 1 we have crest point at t = 10 seconds. That 10 seconds can be split as

t = 10 seconds

t = 2(5) seconds

t = 2 (no. of customers who departed from the system) seconds

t = 2 (1) seconds

Where a is the no. of customers who departed from the system.

The above calculations can be generalized and indicated by the formula.

The curves attaining its maximum height in the neighborhood of t = 2 [N-n]

Most probably at t = 2(N-n) only.

<u>The theoretical proof for the above calculations</u>

From the above calculations the curves are attaining their highest position in the neighborhood of t = 2[N-n] and most probably at t = 2[N-n].

We prove this by assuming the contradiction.

If possible let us suppose at this point, the function is not attaining highest position. That is probability mass accumulated under that point is not more. Since the curve is getting peak point at some position say t. After that curve is descending and then running parallel to x-axis at the end. We must have for some other point (t) the function attains its highest position.

$$\therefore f\left(2\left(N-n\right)\right) < f(t)$$

Where t is other than 2 (N-n) & n is the integer n = 0,1,2, ...N.

Where $f(x)=\dfrac{e^{-\mu x}\left(\mu x\right)^{N-n}\mu}{\left(N-n\right)!}$

$$\frac{e^{-2\mu(N-n)}}{(N-n)!}\left(2\mu\left(N-n\right)\ \right)^{n} < \frac{e^{-\mu t}\left(\mu t\right)^{n}}{n!}$$

$$\frac{e^{-2\mu(N-n)}\left(2\left(N-n\right)^{n}\right)}{n!} < \frac{e^{-\mu t}-t^{n}}{n!}$$

$$e^{-2\mu(N-n)}2^{n}\left(N-n\right)^{n} < e^{-2\mu}t^{n}$$

Case 1:

When t = 0

$$2^{-2\mu(N-n)}2^{n}\left(N-n\right)^{n} < 0$$

This is absurd.

$\therefore f\left(2\left(N-n\right)\right)$ is not less than f(t) when t = 0

Case 2:

When $0 < t < 2(N-n)$

$e^{-2\mu}\left(N-n\right)2^n\left(N-n\right)^n < e^{-2\mu}t^n$

With $e^{-2\mu}\left(N-n\right)$ multiplied by $2^n\left(N-n\right)^n$ cannot be less than $2^{-2\mu}t^n$ with t < 2(N-n)

$\therefore f\left(2\left(N-n\right)\right)$ is not less than f(t) when 0 < t < 2(N-n)

Case 3 :

When t > 2(N-n)

$e^{-2\mu(N-n)}2^n(N-n)^n < e^{-2\mu}t^n$

Due to exponentials on both sides L.H.S. cannot be less than R.H.S. with

t > 2(N-n)

Therefore f(2(N-n)) is not less than f(t) with t > 2(N-n)

Therefore in all the 3 cases f(2(N-n)) is not less than f(t)

f(x) curve is getting its peak position probably at 2(N-n) time where N is restricted number of customers allowed into queuing system and n is the number of persons left in the system after taking (N-n) number of persons their service.

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CONCLUSION

By using the graph of the density function this paper studied some theoretical properties for this density function curves. Using density graphs we developed the relation for the neighborhood of time where these particular curves attain its maximum position in this graph. f(x) curve is getting its peak position probably at 2(N-n) time where N is restricted number of customers allowed into queuing system and n is the number of persons left in the system after taking (N-n) number of persons their service.

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